Technische Elektrodynamik[1]

(Stand 2. Februar 2001)

Allgemeine Vektorrechnung

1 $$\vec{\mathbf{A}}(\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{C}}(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) = \vec{\mathbf{B}}(\vec{\mathbf{C}} \times \vec{\mathbf{A}})$$

2 $$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{B}}(\vec{\mathbf{A}}\vec{\mathbf{C}}) - \vec{\mathbf{C}}(\vec{\mathbf{A}}\vec{\mathbf{B}})$$

3 $$\vec{\mathbf{C}} = \vec{\mathbf{n}}(\vec{\mathbf{n}}\vec{\mathbf{C}}) - \vec{\mathbf{n}} \times (\vec{\mathbf{n}} \times \vec{\mathbf{C}}) \qquad ; \, ||\,\vec{\mathbf{n}}\,|| = 1$$

4 $$(\vec{\mathbf{A}} \times \vec{\mathbf{B}})(\vec{\mathbf{C}} \times \vec{\mathbf{D}}) = (\vec{\mathbf{A}}\vec{\mathbf{C}})(\vec{\mathbf{B}}\vec{\mathbf{D}}) - (\vec{\mathbf{A}}\vec{\mathbf{D}})(\vec{\mathbf{B}}\vec{\mathbf{C}})$$

Kartesische Koordinaten

6 $$\begin{aligned} d\vec{\mathbf{r}} &= \vec{\mathbf{e_x}}\,dx + \vec{\mathbf{e_y}}\,dy + \vec{\mathbf{e_z}}\,dz \\ dv &= dx\,dy\,dz \end{aligned}$$

7 $$\operatorname{grad} V = \vec{\mathbf{e_x}}\frac{\partial V}{\partial x} + \vec{\mathbf{e_y}}\frac{\partial V}{\partial y} + \vec{\mathbf{e_z}}\frac{\partial V}{\partial z}$$

8 $$\operatorname{div} \vec{\mathbf{A}} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

9 $$\operatorname{rot} \vec{\mathbf{A}} = \vec{\mathbf{e_x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{\mathbf{e_y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{\mathbf{e_x}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

10 $$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\left(\vec{\mathbf{A}}\operatorname{grad}\right)\vec{\mathbf{B}} = A_x\frac{\partial \vec{\mathbf{B}}}{\partial x} + A_y\frac{\partial \vec{\mathbf{B}}}{\partial y} + A_z\frac{\partial \vec{\mathbf{B}}}{\partial z}$$

Zylinderkoordinaten

11 $$\begin{aligned} x &= \rho\,\cos\varphi \qquad 0 \le \rho \le \infty \\ y &= \rho\,\sin\varphi \qquad 0 \le \varphi \le 2\pi \\ z &= z \end{aligned}$$

12 $$\begin{aligned} d\vec{\mathbf{r}} &= \vec{\mathbf{e}}_\rho\,d\rho + \vec{\mathbf{e}}_\varphi\rho\,d\varphi + \vec{\mathbf{e_z}}\,dz \\ dv &= \rho\,d\rho\,d\varphi\,dz \end{aligned}$$

13 $$\operatorname{grad} V = \vec{\mathbf{e}}_\rho\frac{\partial V}{\partial \rho} + \vec{\mathbf{e}}_\varphi\frac{1}{\rho}\frac{\partial V}{\partial \varphi} + \vec{\mathbf{e_z}}\frac{\partial V}{\partial z}$$

14 $$\operatorname{div} \vec{\mathbf{A}} = \frac{1}{\rho}\frac{\partial(\rho\,A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

15 $$\begin{aligned} \operatorname{rot} \vec{\mathbf{A}} = \vec{\mathbf{e}}_\rho\left(\frac{1}{\rho}\frac{\partial A_Z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \vec{\mathbf{e}}_\varphi\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \\ \vec{\mathbf{e_z}}\left(\frac{1}{\rho}\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \varphi}\right) \end{aligned}$$

16 $$\Delta V = \frac{\partial^2 V}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial V}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

17 $$\operatorname{rot}\operatorname{rot}(\vec{\mathbf{e}}_\varphi A(\rho, z)) = -\vec{\mathbf{e}}_\varphi\left(\frac{\partial^2 A}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A}{\partial \rho} - \frac{A}{\rho^2} + \frac{\partial^2 A}{\partial z^2}\right)$$

Kugelkoordinaten

18 $$\begin{aligned} x &= \rho\sin\vartheta\cos\varphi \qquad 0 \le \rho \le \infty \\ y &= \rho\sin\vartheta\sin\varphi \qquad 0 \le \vartheta \le \pi \\ z &= \rho\cos\vartheta \qquad 0 \le \varphi \le 2\pi \end{aligned}$$

19 $$\begin{aligned} d\vec{\mathbf{r}} &= \vec{\mathbf{e_r}}\,dr + \vec{\mathbf{e}}_\vartheta\rho\,d\vartheta + \vec{\mathbf{e}}_\varphi\rho\sin\vartheta\,d\varphi \\ dv &= \rho^2\sin\vartheta\,d\rho\,d\vartheta\,d\varphi \end{aligned}$$

20 $$\operatorname{grad} V = \vec{\mathbf{e_r}}\frac{\partial V}{\partial \rho} + \vec{\mathbf{e}}_\vartheta\frac{1}{\rho}\frac{\partial V}{\partial \vartheta} + \vec{\mathbf{e}}_\varphi\frac{1}{\rho\sin\vartheta}\frac{\partial V}{\partial \varphi}$$

21 $$\operatorname{div} \vec{\mathbf{A}} = \frac{1}{\rho^2}\frac{\partial(\rho^2 A_\rho)}{\partial \rho} + \frac{1}{\rho\sin\vartheta}\frac{\partial(A_\vartheta\sin\vartheta)}{\partial \vartheta} + \frac{1}{\rho\sin\vartheta}\frac{\partial A_\varphi}{\partial \varphi}$$

22 $$\begin{aligned} \operatorname{rot} \vec{\mathbf{A}} = \frac{\vec{\mathbf{e_r}}}{\rho\sin\vartheta}\left(\frac{\partial(A_\varphi\sin\vartheta)}{\partial \vartheta} - \frac{\partial A_\vartheta}{\partial \varphi}\right) + \frac{\vec{\mathbf{e}}_\vartheta}{\rho}\left(\frac{1}{\sin\vartheta}\frac{\partial A_\rho}{\partial \varphi} - \frac{\partial(\rho A_\varphi)}{\partial \rho}\right) + \\ \frac{\vec{\mathbf{e}}_\varphi}{\rho}\left(\frac{\partial(\rho\,A_\vartheta)}{\partial \rho} - \frac{\partial A_\rho}{\partial \vartheta}\right) \end{aligned}$$

23 $$\Delta V = \frac{\partial^2 V}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial V}{\partial \rho} + \frac{1}{\rho^2\sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta\,\frac{\partial V}{\partial \vartheta}\right) + \frac{1}{\rho^2\sin^2\vartheta}\frac{\partial^2 V}{\partial \varphi^2}$$

24 $$\begin{aligned} \operatorname{rot}\operatorname{rot}(\vec{\mathbf{e}}_\varphi A(\rho, \vartheta)) = \\ -\vec{\mathbf{e}}_\varphi\left(\frac{\partial^2 A}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial A}{\partial \rho} + \frac{1}{\rho^2\sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta\,\frac{\partial A}{\partial \vartheta}\right) - \frac{A}{\rho^2\sin^2\vartheta}\right) \end{aligned}$$

Umformungen von grad

25 $$\operatorname{grad}(\psi_1\psi_2) = \psi_1\operatorname{grad}\psi_2 + \psi_2\operatorname{grad}\psi_1$$

26 $$\operatorname{grad}(\vec{\mathbf{A}}\,\vec{\mathbf{B}}) = \left(\vec{\mathbf{A}}\operatorname{grad}\right)\vec{\mathbf{B}} + \left(\vec{\mathbf{B}}\operatorname{grad}\right)\vec{\mathbf{A}} + \vec{\mathbf{A}} \times \operatorname{rot}\vec{\mathbf{B}} + \vec{\mathbf{B}} \times \operatorname{rot}\vec{\mathbf{A}}\}$$

27 $$\operatorname{grad}\,\vec{\mathbf{n}}\,\vec{\mathbf{r}} = \vec{\mathbf{n}} \qquad ; \vec{\mathbf{n}} \text{ konstant}$$

[1] Abgetippt von der Formelsammlung zur Vorlesung Technische Elektrodynamik von Prof. Dr.-Ing. M. Albach. Gab nichts zu verbessern, ausser das Ganze — leider auf Kosten der Lesbarkeit — auf 2 Seiten zu quetschen, und um die Maxwell'schen Gleichungen zu ergänzen. Fehler bitte an mich melden. Gunter Königsmann ⟨gunter.koenigsmann@gmx.de⟩

28 $\operatorname{grad}(\psi(r)) = \frac{\vec{\mathbf{r}}}{r}\frac{\partial\psi}{\partial r} \leadsto \operatorname{grad}\frac{1}{r} = -\frac{\vec{\mathbf{r}}}{r^3}$

29 $d\vec{\mathbf{r}}\operatorname{grad}\psi = d\psi$

$\left(\vec{\mathbf{A}}\operatorname{grad}\right)\vec{\mathbf{B}} = \vec{\mathbf{A}}\frac{\partial(B_1, B_2,\ B_3)}{\partial(e_1,\ e_2,\ e_3)} = |\vec{\mathbf{A}}|\frac{d\vec{\mathbf{B}}}{d\left(\frac{\vec{\mathbf{A}}}{|\vec{\mathbf{A}}|}\right)} = |\vec{\mathbf{A}}|^2\frac{d\vec{\mathbf{B}}}{d\vec{\mathbf{A}}}$

30 $\left(\vec{\mathbf{A}}\operatorname{grad}\right)\vec{\mathbf{r}} = \vec{\mathbf{A}}$

Umformungen von div

31 $\operatorname{div}(\psi\vec{\mathbf{A}}) = \psi\operatorname{div}\vec{\mathbf{A}} + \vec{\mathbf{A}}\operatorname{grad}\psi$

32 $\operatorname{div}(\vec{\mathbf{A}}\times\vec{\mathbf{B}}) = \vec{\mathbf{B}}\operatorname{rot}\vec{\mathbf{A}} - \vec{\mathbf{A}}\operatorname{rot}\vec{\mathbf{B}}$

33 $\operatorname{div}(\vec{\mathbf{r}}\psi(r)) = 3\psi(r) + r\frac{\partial\psi}{\partial r}$

34 $\operatorname{div}\operatorname{grad}\psi = \Delta\psi$

35 $\operatorname{div}\operatorname{rot}\vec{\mathbf{A}} = 0$

Umformungen von rot

36 $\operatorname{rot}(\psi\vec{\mathbf{A}}) = \psi\operatorname{rot}\vec{\mathbf{A}} - \vec{\mathbf{A}}\times\operatorname{grad}\psi$

37 $\operatorname{rot}\left(\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right) = \left(\vec{\mathbf{B}}\operatorname{grad}\right)\vec{\mathbf{A}} - \left(\vec{\mathbf{A}}\operatorname{grad}\right)\vec{\mathbf{B}} + \vec{\mathbf{A}}\operatorname{div}\vec{\mathbf{B}} - \vec{\mathbf{B}}\operatorname{div}\vec{\mathbf{A}}$

38 $\operatorname{rot}\vec{\mathbf{r}} = 0$

39 $\operatorname{rot}(\vec{\mathbf{n}}\times\vec{\mathbf{r}}) = 2\vec{\mathbf{n}}$; $\vec{\mathbf{n}}$ konstant

40 $\operatorname{rot}\operatorname{grad}\psi = 0$

41 $\operatorname{rot}\operatorname{rot}\vec{\mathbf{A}} = \operatorname{grad}\operatorname{div}\vec{\mathbf{A}} - \Delta\vec{\mathbf{A}}$

42 $\operatorname{rot}(\Delta\vec{\mathbf{A}}) = \Delta(\operatorname{rot}\vec{\mathbf{A}})$

43 $\operatorname{rot}\operatorname{rot}\operatorname{rot}(\vec{\mathbf{n}}\psi) = \vec{\mathbf{n}}\times\operatorname{grad}(\Delta\psi)$; $\vec{\mathbf{n}}$ konstant

44 $\operatorname{rot}\operatorname{rot}\operatorname{rot}(\vec{\mathbf{r}}\psi) = \vec{\mathbf{r}}\times\operatorname{grad}(\Delta\psi)$

Umformungen von Δ

45 $\Delta V = \operatorname{div}\operatorname{grad}V$

46 $\Delta(\vec{\mathbf{r}}\psi) = 2\operatorname{grad}\psi + \vec{\mathbf{r}}\Delta\psi$

47 $\Delta\vec{\mathbf{A}} = \vec{\mathbf{e_x}}\Delta A_x + \vec{\mathbf{e_y}}\Delta A_y + \vec{\mathbf{e_z}}\Delta A_z$

Integralsätze

48 $\iint_F \operatorname{rot}\vec{\mathbf{A}}\,d\vec{\mathbf{F}} = \oint_C \vec{\mathbf{A}}\,d\vec{\mathbf{s}}$ Stokes

49 $\iint_F \operatorname{grad}\psi\times d\vec{\mathbf{F}} = -\oint_C \psi\,d\vec{\mathbf{s}}$ Stokes, modifiziert

50 $\iiint_V \operatorname{div}\vec{\mathbf{A}}\,dv = \oiint_F \vec{\mathbf{A}}\,d\vec{\mathbf{F}}$ Gauss

51 $\iiint_V \operatorname{grad}\psi\,dv = \oiint_F \psi\,d\vec{\mathbf{F}}$ Gauss, modifiziert

52 $\iiint_V \operatorname{rot}\vec{\mathbf{A}}\,dv = -\oiint_F \vec{\mathbf{A}}\times d\vec{\mathbf{F}}$

53 $\iiint_V (\psi_1\Delta\psi_2 + \operatorname{grad}\psi_1\ \operatorname{grad}\psi_2)\,dv = \oiint_F \psi_1\ \operatorname{grad}\psi_2\,d\vec{\mathbf{F}}$ Green I

54 $\iiint_V (\psi_1\Delta\psi_2 - \psi_2\Delta\psi_1)dv = \oiint_F (\psi_1\ \operatorname{grad}\psi_2 - \psi_2\operatorname{grad}\psi_1)\,d\vec{\mathbf{F}}$ Green II

55 $\iiint_V \Delta\psi\,dv = \oiint_F \operatorname{grad}\psi\,d\vec{\mathbf{F}}$

56 $\iiint_V \operatorname{rot}\vec{\mathbf{A}}\ \operatorname{rot}\vec{\mathbf{B}} - \vec{\mathbf{A}}\operatorname{rot}\operatorname{rot}\vec{\mathbf{B}})dv = \oiint_F (\vec{\mathbf{A}}\times\operatorname{rot}\vec{\mathbf{B}})d\vec{\mathbf{F}}$ Stratton I

57 $\iiint_V (\vec{\mathbf{A}}\operatorname{rot}\operatorname{rot}\vec{\mathbf{B}} - \vec{\mathbf{B}}\operatorname{rot}\operatorname{rot}\vec{\mathbf{A}})dv = \oiint_F (\vec{\mathbf{B}}\times\operatorname{rot}\vec{\mathbf{A}} - \vec{\mathbf{A}}\times\operatorname{rot}\vec{\mathbf{B}})d\vec{\mathbf{F}}$ Stratton II

$\oint_C \operatorname{grad}\psi\,d\vec{\mathbf{s}} = 0$

58

$\oiint_F \operatorname{rot}\vec{\mathbf{A}}\,d\vec{\mathbf{F}} = 0$

Die Maxwell'schen Gleichungen[2]

$\oint_C \vec{\mathbf{H}}(t)\,d\vec{\mathbf{s}} = \iint_F \vec{\mathbf{J}}\,d\vec{\mathbf{F}} + \frac{\partial}{\partial t}\iint_F \vec{\mathbf{D}}(t)\,d\vec{\mathbf{F}}$ $\qquad \nabla\times\vec{\mathbf{E}}(t) = -\frac{\partial}{\partial t}B(t)$

$\oint_C \vec{\mathbf{E}}(t)\,d\vec{\mathbf{s}} = -\frac{\partial}{\partial t}\iint_F \vec{\mathbf{B}}(t)\,d\vec{\mathbf{F}}$ $\qquad \nabla\times\vec{\mathbf{H}}(t) = \vec{\mathbf{J}}(t) + \frac{\partial}{\partial t}\vec{\mathbf{D}}(t)$

$\oiint_F \vec{\mathbf{D}}(t)\,d\vec{\mathbf{F}} = \iiint_V \varrho(t)\,dv$ $\qquad \nabla\vec{\mathbf{D}}(t) = \varrho(t)$

$\oiint_F \vec{\mathbf{B}}(t)\,d\vec{\mathbf{F}} = 0$ $\qquad \nabla\vec{\mathbf{B}}(t) = 0$

$\vec{\mathbf{B}} = \operatorname{rot}\vec{\mathbf{A}}$ $\qquad \vec{\mathbf{D}}(t) = \varepsilon\vec{\mathbf{E}}(t)$

$\vec{\mathbf{E}} = -\operatorname{grad}\varphi$ $\qquad \vec{\mathbf{B}}(t) = \mu\vec{\mathbf{H}}(t)$

$\vec{\mathbf{H}} = -\operatorname{grad}V$; $\vec{\mathbf{J}} = 0$ $\qquad \vec{\mathbf{J}}(t) = \sigma\vec{\mathbf{E}}(t)$

[2] Abgetippt vom BEL III- Skriptum von Prof. Dr. Ing. L.-P. Schmidt, Seite 33